EE2030010: Linear Algebra **Spring 2018**
Department of Electrical Engineering
National Tsing Hua University

Homework #1
Coverage: chapter 1-2
Due date: 23 March, 2018

Instructor: Chong-Yung Chi *TAs: Amin Jalili & Yao-Rong Syu*

Notice:
1. Please hand in your answer sheets by yourself before 23:59 of the due date. No late homework will be accepted.
2. Please justify your answers with clear, logical and solid reasoning or proofs.
3. Please do the homework independently by yourself. However, you may discuss with someone else but copyied homework is not allowed. This will show your respect toward the academic integrity.

Problem 1. (10 points) Consider the vectors $\mathbf{a}_1 = [\,2 \quad 3 \quad 5\,]^T$, $\mathbf{a}_2 = [\,6 \quad 15 \quad 9\,]^T$ and $\mathbf{a}_3 = [\,0 \quad -2 \quad 2\,]^T$.

(i) Are these three vectors independent?

(ii) Determine whether the matrix defined as $\mathbf{A} \triangleq [\,\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3\,]$ is invertible or not.

Problem 2. (15 points) Let $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{A}+\mathbf{B} \in \mathbb{R}^{n\times n}$ be nonsingular matrices. Show that:

$$\mathbf{A}^{-1}(\mathbf{A}^{-1}+\mathbf{B}^{-1})^{-1}\mathbf{B}^{-1} = (\mathbf{A}+\mathbf{B})^{-1}.$$

Problem 3. (10 points) Consider the linear equations $x+y+2z=1$ and $-x+3y+2z=3$. Find a third equation such that there is no intersection point for the system of these three equations.

Problem 4. (15 points) Consider the matrix $\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix}$. Using elimination steps find the matrix $\mathbf{E}$ such that $\mathbf{EP} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix.

Problem 5. (20 points) Consider the matrix $\mathbf{A} = \begin{bmatrix} 1 & a & 0 & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Find the inverse of $\mathbf{A}$ using the Gauss-Jordan elimination method.

Problem 6. (20 points) Consider $\mathbf{K} = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 3 & -1 \\ -6 & -2 & a \end{bmatrix}$ where $a \in \mathbb{R}$.

(i) Compute the LU factorizaion of the matrix $\mathbf{K}$.

(ii) Find condition on a such that LU has three pivots.

Problem 7. (10 points) Let $\mathbf{v}_1 = \begin{bmatrix} \alpha-3 \\ 2+\alpha \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2-\alpha \\ \alpha+3 \end{bmatrix}$ where $\alpha \in \mathbb{R}$. Find the values of α such that $\mathbf{v}_1$ and $\mathbf{v}_2$ be linearly independent.